A New Way to Set the Maximum Weight in the Weighting Algorithm of UTC

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***Summary*—In this paper a new way to set the maximum weight in the weighting algorithm of UTC based on the Karush-Kuhn-Tucker conditions is presented. The idea is to impend several clocks (H-masers) reaching the maximum weight if they are not good enough. This is due to the current way to fix the maximum weight and to the normalization process applied. The final scope is the improvement of long-term stability of UTC.**

Keywords—UTC, weighting algorithm, long-term stability.

I. INTRODUCTION

UTC is calculated with the procedure reported in [2]. One of the most important algorithms used in UTC is the weighting algorithm (the detailed steps are reported here [3]). The role of this algorithm is to give a weight to the clocks to ensure the long-term stability to UTC by assuring its reliability. Briefly, the algorithm can be resumed as an iterative procedure where the weight is calculated as the inverse of a statical estimator (variances, Allan variances etc.) and subject to the constraint of a maximum weight. The problem is solved by using Lagrange multiplier method. From the theoretical point of view an optimal solution is found if not constraints are imposed to the system. However, as the reliability of UTC is a mandatory condition a maximum weight is fixed. In this way we guarantee that a good number of clocks are always at maximum weight in UTC even if the ensemble changes continuously.

In [1] a study has been conducted to statistically characterize the atomic clocks (H-Masers in particular) participating in UTC. This study showed that H-Masers have different level of stability and predictability primarily depending on the aging. In general, it has been found that the newer H-masers are more efficient than the old ones although the latter are still considerably better than the cesium clocks. We also checked how these clocks are weighted in UTC and, with the current value imposed for the maximum weight (4 / N, N is the total number of clocks), these clocks are all at the maximum weight in UTC. The algorithm is not able to distinguish between these two groups of clocks.

After testing different values for the maximum weight and concluding that a different choice of 4/N should be made, a completely new approach was considered.

By using the Karush–Kuhn–Tucker conditions [4], it is possible to generalize the maximum weight constraints in the system which is solved to obtain the weights of the clocks.

In this article, the use of 2 maximum weights (instead of one) is tested to allow the algorithm to distinguish between the two categories of H-Masers.

This new approach succeeds in better weighting the clocks and shows that a significant improvement in the long-term stability of UTC can be obtained.

II. METHODS

Applying the Karush–Kuhn–Tucker conditions means to solve a system with constraints. To improve the long term stability of UTC (that means to optimize the calculation of EAL the free atomic time scale described in [1]) the functional to be minimized is the standard deviation of EAL:

$$\min \sigma(EAL) = \min \sigma\left(\sum_{i=1}^{N} w_i \times r_i\right) \quad (1)$$

where w_i are the weights and $r_i = h_i - h_i'$ are the difference between the clocks and their predictions. The minimization of the standard deviation of (1) is subject to the constraints:

$$\begin{cases} \sum_{i=1}^{N} w_i = 1 \\ w_i \le w_{MAX} \ \forall i \\ w_i \ge 0 \quad \forall i \end{cases} \quad (2)$$

The constraints (2) can be generalized as for example using 2 or more w_{MAX}.

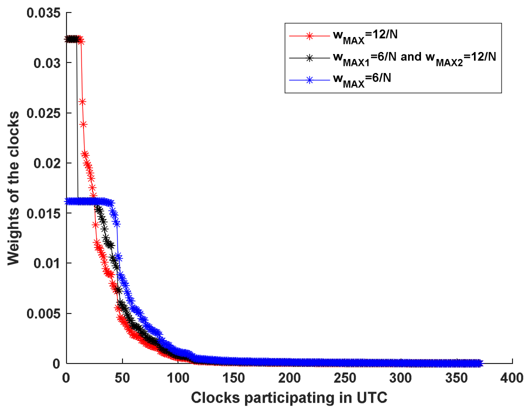
One month of UTC calculation (with N=433 of total atomic clocks and 170 H-Masers) has been considered with the weights calculate by the current used weighting algorithm.

By solving the system (1) and (2) with different values for the maximum weights new sets of weights have been calculated. The standard deviation of the weighted residuals (EAL) was evaluated for these different cases to compare the obtained results.

III. RESULTS

Figure (1) shows the different sets of weights obtained by the algorithm and used for the weighted residuals. The data are listed in decreasing order to highlight the number of clocks at maximum weight. With the blue line the data obtained with the maximum weight equal to 6/N, with the red line 12/N and with the black line the two maximum weights are reported. Information on the number of clocks at maximum weight is important because, to guarantee reliability to UTC, a stable set of clocks is required. This prevents UTC stability from being affected by changes in a small group of clocks if they are at maximum weight. Table I contains the number of clocks with maximum weight corresponding to the scenarios described above. Also, the current situation (maximum weight equal to 4/N) is reported in Table I.

Figure 1 Weights obtained by imposing the maximum weights as indicated in the legend of the figure.



By analyzing the data reported in Table I we can conclude that the number of clocks at maximum weights are:

- 12, by fixing the maximum weight equal to 12/N. A very small number to guarantee the reliability of UTC
- 58, by fixing the maximum weight equal to 4/N (current situation), represents about 13% of the total clocks and 30% of the H-masers. A too big number.
- 35, by fixing 2 maximum clocks, represents about 0.08% of the total clocks and 20% of the H masers. Probably the right number required.

TABLE I. NUMBER OF CLOCKS AT MAXIMUM WEIGHT DEPENDING ON THE SETTED MAXIMUM WEIGHT

Maximum weights with N=433	**Number of clocks at maximum weight**
W_{MAX}=4/N	58
W_{MAX}=6/N	33
W_{MAX}=12/N	12
W_{MAX1}=6/N W_{MAX2}=12/N	35

With these set of weights, we have evaluated the standard deviation of the residuals for the whole calculation month of UTC.

Figure (2) reports the standard deviations obtained with different sets of weights as indicated in the legend of the figure. The results are obtained by fixing a single maximum weight from 4/N to 14/N and 2 maximum weights as 12/N and 6/N, 12/N and 8/N and finally 14/N and 8/N.

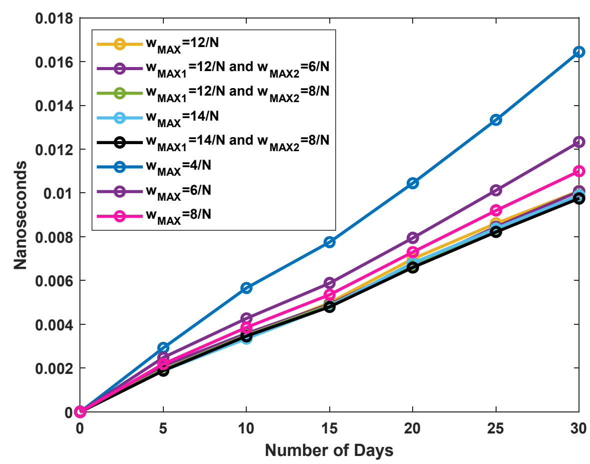
By analyzing the results, two major conclusions can be raised:

- an important improvement in the standard deviation is observed if a bigger maximum weight is applied (as expected by the theory)
- the use of Karush–Kuhn–Tucker conditions, allowing the use of two maximum weights, allows very low values for the standard deviation to be achieved maintaining an acceptable number of clocks at maximum weight.

This result is very encouraging for improving the long-term stability of UTC and for adequately handling the clocks participating in the calculation of UTC.

Repetitive tests over a longer period are suitable to consider all the eventualities that may happen to UTC (unexpected changes in the ensemble, time, and frequency steps etc.) and to adapt the algorithm accordingly.

Figure 2 Standard deviation of the weighted residuals calculated by setting the maximum weights as indicated in the legend of the figure.



IV. CONCLUSIONS

In this paper a new method to set the maximum weight was presented. The very good results obtained show a new way to improve the long-term stability of UTC.

REFERENCES

[1] J. Milton and G. Panfilo Statistical analysis of H-Masers to improve the long-term stability of UTC. Proceedings of EFTF-IFCS June 2021.

[2] Panfilo G and Arias F 2019 The Coordinated Universal Time (UTC) *Metrologia* **56** 042001.

[3] Panfilo G, Harmegnies A, Tisserand L 2014 A new weighting procedure for UTC *Metrologia* **51** 285.

[4] Solving Least Square Problems, Charles L. Lawson and Richard J. Hanson. Classics in applied mathematics. SIAM.